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On the Relationship of the Scaled Phase Space and Skyrme-Coherent State Treatments of Proton Antiproton Annihilation at Rest

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Abstract

We discuss pion multiplicities and single pion momentum spectra from proton antiproton annihilation at rest. Both the scaled phase space model and the Skyrme-coherent state approach describe these observables well. In the coherent state approach the puzzling size of the scale parameter relating the phase space integrals for different multiplicities is replaced by a well defined weight function. The strength of this function is determined by the intensity of the classical pion field and its spatial extent is of order 1 fm.

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The phenomenology of proton antiproton annihilation at rest is a venerable subject going back to Fermi [1]. The connection of successful phenomenology with dynamical theory is difficult because low energy annihilation into pions is squarely in the domain of nonperturbative QCD. Recently we have shown that Skyrmion dynamics, a model based on a classical approach to QCD, combined with coherent states to account for the quantum nature of the fields, gives a remarkably good account of many of the features of low energy annihilation and does so with essentially no free parameters [2]. In this note we compare the Skyrme-coherent state approach with the scaled form of the statistical phase space method introduced by Fermi. In particular we explore what parts of these approaches are responsible for the general agreement, what parts are needed for the details, and what parts serve to bridge the gap from phenomenology to dynamical model.

1 Scaled Phase Space

The most important constraint on any calculation of pions from proton-antiproton annihilation at rest is that of energy momentum conservation. That is, the rate for finding n pions of three-momenta $\mathbf{k}_i$, with $i = 1...n$, should be proportional to the differential phase space factor $\rho_n(s, \{\mathbf{k}_i\})$ given by

$$\rho_n(s, \{\mathbf{k}_i\}) = \delta^4(k_t - \sum_{i=1}^{n} k_i) \prod_{i=2}^{n} \frac{d^3\mathbf{k}_i}{2\omega_i} \tag{1}$$

where $s = (k_t)^2$, k_t is the total four-momentum of the annihilating pair and ω_i is the energy of the i-th pion, $\omega_i = \sqrt{\mathbf{k}_i^2 + \mu^2}$, with μ the pion mass. For annihilation at rest $k_t = (2M, \mathbf{0})$, with M the nucleon mass. The "phase space only" (PSO) assumption is that all other aspects of the annihilation process depend very weakly on $\{\mathbf{k}_i\}$ so that the entire dependence is given by $\rho_n(s, \{\mathbf{k}_i\})$. For example the single pion momentum spectrum for n pions is obtained by integrating $\rho_n(s, \{\mathbf{k}_i\})$ over all but one of the final momenta. We will return below to the result of that integration.

If we want to compare the branching ratio for n to that for $n + 1$ pions we observe that the corresponding integrals in (1) differ in dimension by two units of momentum. Thus to compare them, all other things being equal, we must construct a quantity of uniform dimension, $R(n)$, by scaling the total

phase space for n pions we write

$$R(n) = \frac{(L)^{2n}}{n!} \int \rho_n(s, \{\mathbf{k}_i\}) \tag{2}$$

where L is a length and an n independent overall normalization factor is set equal to 1. We call this picture for relating multiplicities by a dimensional scaling "scaled phase space," SPS. In terms of $R(n)$ one can calculate the average number of pions in the SPS picture by

$$\hat{n} = \frac{\sum_n nR(n)}{\sum_n R(n)} \tag{3}$$

This $\hat{n}$ depends on L which can be varied to give the experimental value, $\hat{n} = 5$. With the scaling length fixed by this empirical constraint, the resulting pion multiplicity distribution, the normalized $R(n)$, looks very much like the experimental one. Fig. 1 shows the pion multiplicity distribution calculated in the SPS formalism[2] with $L = 1.2$ fm, adjusted to give the correct average pion number. It has a gaussian shape with an average of 5 (put in by hand) and a variance of 1. Note no statistical assumption has been made to obtain the observed gaussian distribution or the correct variance. Fig. 1 also displays the corresponding Skyrme-coherent state calculation of Sect. 2 and the measured distribution. Both calculations agree excellently with the data.

In earlier treatments of the scaling model L was connected to a volume [3, 5] by $V = (2\pi L)^3$. The 2π comes from the normalization of phase space density. The volume needed to give $\hat{n} = 5$ turns out to be very large by nucleon standards, of order $(2\pi)^3$ fm^3. There is much discussion in the early literature about the meaning of this large volume [3, 5]. An alternate picture is that one should write $V = g^2 \cdot v$ where v is a reasonable volume ($\sim \frac{4\pi}{3}\mu^{-3}$ $= 12.5$ fm^3) while g is a dimensionless number that gives the amplitude for emitting one more pion. Now it is g that is large. The notion that the amplitude for emitting n pions should be proportional to g^n, the coupling constant to the n-th power, makes sense perturbatively, but is being employed here for g large. Annihilation is nonperturbative. Thus the dynamical origin of the SPS picture remains obscure, even though it can certainly fit the pion multiplicity distribution with one free parameter.

[2] The non-relativistic form of phase space leads to very similar distributions, as we have checked and as was already reported in [3]

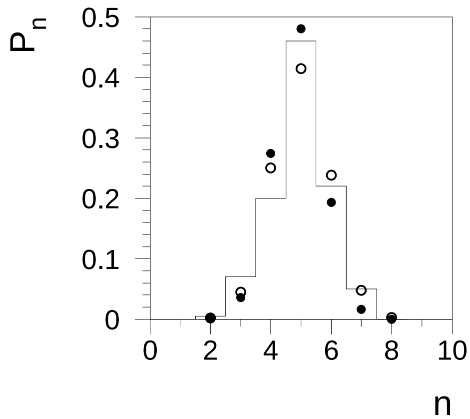

Figure 1: Pion multiplicity distribution in proton antiproton annihilation at rest. The open dots are from the scaled phase space model and the solid dots the from Skyrme-coherent state approach. Both have been fixed to give an average pion multiplicity of 5. The histogram is the data presented in [3, 4].

Historically statistical approaches were unable to account for the pion multiplicity distribution in a pions only picture with a "reasonable" scaling parameter, or to connect the picture to more fundamental theory. Adding heavy mesons that decay sequentially to pions helps the phenomenology, but at the expense of more free parameters [6]. There have also been attempts to make thermodynamic models of annihilation without imposing the constraints of energy-momentum conservation [7]. These do poorly for the pion multiplicity distribution even when $\hat{n}$ is fixed at 5. This serves to further emphasize the primary importance of the four-momentum phase space constraint.

For fixed multiplicity, n, the PSO picture has no dynamics. Relating the amplitude for one n to the next implies dynamical assumptions outside PSO as in the scaled phase space scheme SPS, discussed above. For a natural candidate that introduces genuine dynamics we turn to the Skyrme-coherent state approach. It relates the weights and momentum spectra of n pion emission in a transparent way to the properties of the underlying annihilation process.

2 Skyrme-Coherent State Approach

In the Skyrme-coherent state approach one uses a classical meson field theory in which baryons appear as topologically stable solitons to model the dynamics of annihilation. This picture, invented by T.H.R. Skyrme [8], is connected to QCD in the limit of a large number of colors, and through that to the long wave length or nonperturbative limit of QCD [9, 10]. It is found that in the Skyrme approach, annihilation proceeds very rapidly leading to a burst of relatively intense classical pion radiation [11, 12]. To connect with the physical pion quanta of experiment, that classical wave is used to generate a quantum mechanical coherent state [13]. There are two steps to this process, the use of Skyrme dynamics to generate the classical pion radiation from annihilation, and the subsequent quantization of that radiation using coherent states. The circumstances of annihilation seem particularly well suited to this combination.

A standard coherent state does not have fixed four-momentum, but as we saw above, that constraint is crucial. Hence the coherent state must be projected onto a state of definite energy-momentum [14], and if we are interested in pion charge ratios, a state of definite isospin [15]. We have developed the formalism for doing all this [2]. A pion coherent state contains an exponential in the pion field creation operator. It is a single quantum state containing all pion numbers. This is the physics appeal of the coherent state approach, namely that all the pion channels are collected into a single state. Thus questions about the relation of the rate or spectrum in the n pion channel to that in the $n+1$ channel, are naturally answered. No new parameters or assumptions are needed to address them. Since the coherent state approach can also be generalized to include energy-momentum conservation, the results discussed in Sect. 1 come out, but now with a clear origin for the relationship among the channels.

In the Skyrme-coherent state approach, the difficult dynamics of nucleon-antinucleon interaction and subsequent annihilation into pions is done classically, and quantum mechanics only enters to describe the propagation of the coherent state after the fields have reached the radiation zone, where they are non-interacting. Although this program is far simpler than the corresponding full quantum QCD calculation of annihilation, it is still complicated to execute with the classical, nonlinear field equations of Skyrme and up to now has not been fully implemented. We have not studied the development of

the annihilation process itself, but rather have begun with an assumed initial spherically symmetric configuration of classical pion field. It is in this initial pion configuration, that free parameters enter. The remaining dynamics is completely determined. We take a simple initial pion configuration characterized by a size and magnitude. The magnitude is fixed by the total energy of the system, $2M$, leaving only the size to be determined. This is fit to the average pion multiplicity, or equivalently, the inclusive single pion momentum distribution. We find a size of order 1 fm, a completely reasonable result. Note that if at some future date we are able to do the Skyrme calculation of annihilation from the beginning, there would be no free parameters whatsoever.

The introduction of a finite source size for the pion radiation leads to a form factor for pion emission $f(k_i)$. This form factor is the Fourier transform of the classical asymptotic pion field. As such it is similar to, but not identical to the Fourier transform of the pion source distribution because of the intervening Skyrmion dynamics. For the coherent state formalism of [2] we must replace (2) by

$$R^{COH}(n) = \frac{1}{n!} \int \prod_{i=2}^{n} |f(k_i)|^2 \rho_n(s, \{\mathbf{k}_i\}) \tag{4}$$

(without the complication of isospin). This expression is completely specified by the coherent state formalism through the form factors. Hence in the Skyrme-coherent state approach the ratios of multiplicities are controlled not by some arbitrary volume, but by the intensity of the classical radiation field. In this work we take the analytic form for the form factor which we have used before [2]. Adapted to the relativistic phase space convention it reads

$$|f(k)|^2 = \frac{2C_0 \mathbf{k}^2}{\omega(\mathbf{k}^2 + \alpha^2)^2(\omega^2 + \alpha^2)^2} \tag{5}$$

with $\alpha = 3.0\mu$ and $C_0 = 0.061$ GeV^5. C_0 has been determined by requiring that the energy release of the classical pion source is $2M$. The size parameter α has been adjusted to the measured average multiplicity $\hat{n} = 5$ imposing energy momentum-conservation[3] as in [2].

[3]Without energy momentum conservation the classical mean pion multiplicity $\int |f(k)|^2 d^3\mathbf{k}(2\omega)^{-1}$ is 4.5 for $\alpha = 3\mu$.

Note that any classical field theory capable of describing the evolution of classical pion radiation from annihilation will lead to a quantum coherent state and to form factors. We emphasize the Skyrme method because it is the only one we know that naturally gives annihilation and subsequent pion radiation.

For the pion multiplicity spectrum we have seen in Fig. 1 that the introduction of form factors in the coherent state approach leads to results equivalent to those of the SPS model and in agreement with experiment. However now the relative weighting of different multiplicities has a simple physical origin in the strength and size of the initial pion source. The strength is determined by the energy release and the spatial distribution corresponding to (5) has an r.m.s. of 0.7 fm which is reasonable. Next we calculate the single pion momentum spectra for the pion multiplicities individually, by integrating the phase space over all but one of the pion momenta

$$\begin{aligned} \frac{dN_n(K)}{dK} &= \frac{1}{n!}\int \delta(K-|K_1|)\prod_{i=2}^{n}|f(k_i)|^2\rho_n(s,\{\mathbf{k}_i\}) \\ \frac{dN(K)}{dK} &= \sum_n \frac{dN_n(K)}{dK} \end{aligned} \tag{6}$$

where we use the notation $K = |\mathbf{k}|$.

In Fig. 2 we show the single particle spectra (dN_n/dK) for $n = 3$ to 8 final pions for the scaled phase space model SPS and for the coherent state approach [2, 16]. For each n, each graph is normalized to 1. We see that there is some difference in detail between the SPS and the coherent state approach, but they are quite similar in general shape. We have been unable to find recent data with which to compare these pion spectra for fixed n. In Fig. 3 we show the single pion momentum spectra of Fig. 2 with their correct relative weights. The probabilities of the different multiplicities, P_n are listed on the figure. For the coherent state case the relative weights come out of the dynamics, for the phase space case they are put in through the scaling, fit to give $\hat{n} = 5$. We see that in both cases only the $n = 4, 5, 6$ multiplicities have substantial weight. Also shown in Fig. 3 are the weighted sum (normalized to one) of the momentum distributions in each multiplicity, the inclusive single pion momentum spectrum. In Fig. 4 we show that inclusive pion spectrum again (this time normalized to the total number of direct pions) comparing the SPS result, the coherent state result and the data reported in [3]. The

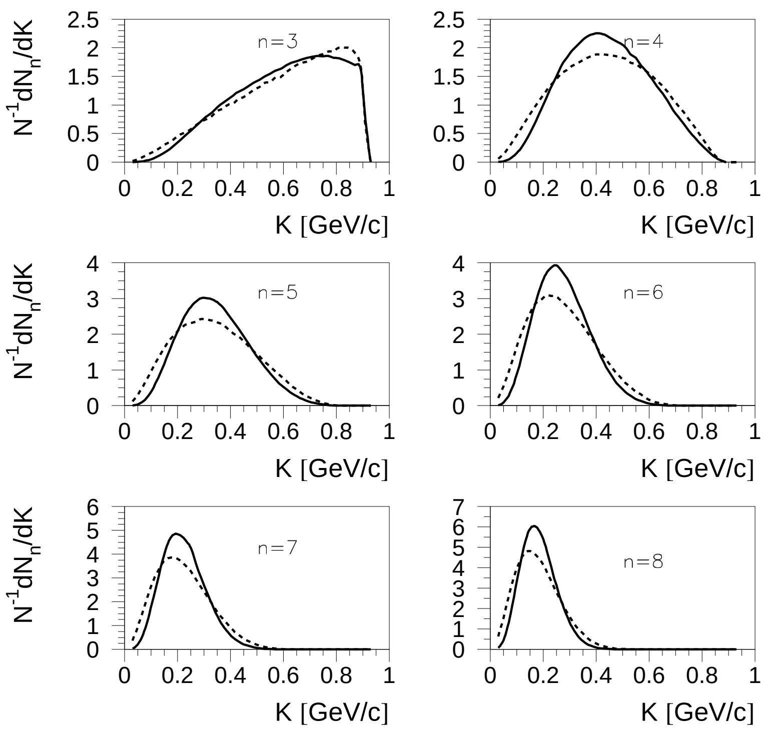

Figure 2: The single pion inclusive momentum distribution (dN_n/dK) for annihilation to channels with pion multiplicity $n = 3$ to 8. The solid line is from the Skyrme-coherent state approach and the dotted line is phase space only. All distributions are normalized to one.

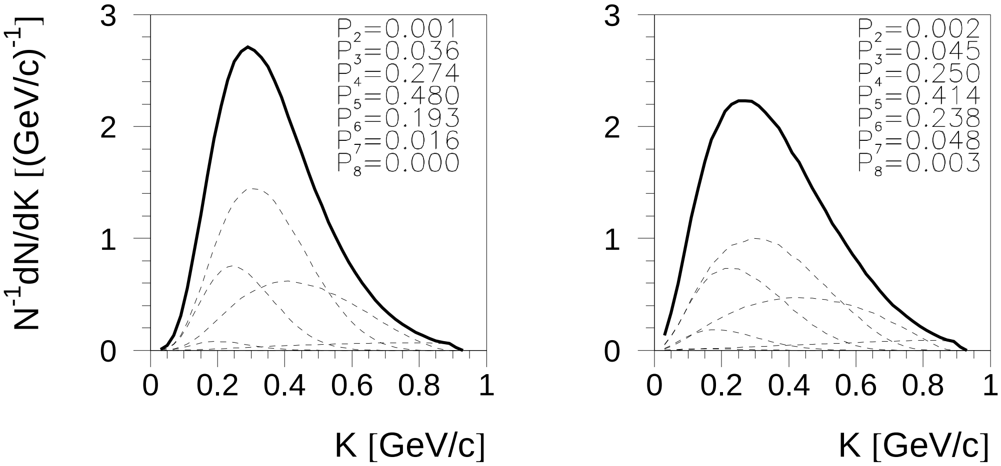

Figure 3: The single pion inclusive momentum distributions dN_n/dK for each multiplicity weighted by the probability of that multiplicity and the sum of these, dN/dK, which is the full inclusive pion momentum spectrum. The Skyrme-coherent state case is shown on the left and the scaled phase space case on the right. The probabilities of the different multiplicities are shown on the graph. The summed spectrum is normalized to one.

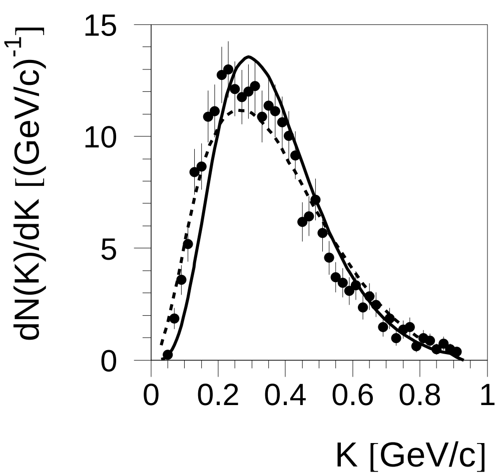

Figure 4: The single pion momentum distribution, dN/dK, summed over all multiplicities, normalized to the total number of pions. The dotted line is from the scaled phase space model, the solid line from the Skyrme-coherent state approach and the data are from [4].

two calculations agree qualitatively, giving an equivalently good account of the data.

3 Conclusions

We have seen that the principal features of the pion multiplicity distribution and of the pion momentum spectrum in proton antiproton annihilation at rest can come from phase space so long as one connects the probabilities for different pion multiplicities. This can be done in an ad hoc way in the scaled phase space picture by introducing a scaling volume or in a dynamically motivated way in the context of the Skyrme-coherent state approach. The scaling volume needed in the phase space picture, $(2\pi)^3$ fm^3, is an order of magnitude too large. No such volume interpretation is required in the Skyrme approach. Rather a form factor appears naturally the strength of which is fixed by the magnitude of the classical pion field, or equivalently by the energy released in annihilation, $2M$, and the range of which is fit to get an average pion number of 5, yielding a size of about 0.7 fm. Furthermore if a complete calculation of annihilation using Skyrme dynamics were carried out (a difficult but not impossible task) there would be no free parameters in its description of annihilation.

Finally we should point out that we have only discussed inclusive pion multiplicity and momentum spectra here. The Skyrme-coherent state picture has also been used to calculated pion charge branching ratios and extended to include vector mesons all with no new free parameters [16]. These vector mesons are generated by the extended Skyrme dynamics since we take the initial configuration to be pions only. The calculated charge branching ratios and branching ratios into vector mesons (ρ and ω) are in qualitative agreement with experiment [3, 4]. A corresponding phase space only calculation would require additional free parameters to generate these branching ratios. Further afield, two pion correlations, which have been discussed in the Skyrme-coherent state picture [17, 18], find no natural explanation in the scaled phase space approach.

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References

[1] Fermi, E.: Prog. Theor. Phys. **5**, 570(1950)
[2] Amado, R.D., Cannata, F., Dedonder, J-P., Locher, M.P., Shao, B.: Phys. Rev. Lett. **72**, 970(1994); Phys. Rev. **C50**, 640(1994)
[3] Dover, C.B., Gutsche, T., Maruyama, M., Faessler, A.: Prog. Part. Nucl. Phys. **29**, 87(1992)
[4] Sedlák, J., Šimák, V.: Sov. J. Part. Nucl. **19**, 191(1988)
[5] Kretzschmar, M.: Ann. Rev. Nucl. Sci. **11**, 1(1961)
[6] Vandermeulen, J.: Z. Phys. **A342**, 329(1992)
[7] Blümel, W., Heinz, U.: Z. Phys. **C67**, 281(1995)
[8] Skyrme, T.H.R.: Proc. R. Soc. London **262**, 237(1961); Nucl. Phys. **31**, 556(1962)
[9] 't Hooft, G.: Nucl. Phys **B72** 461(1974); **B75**, 461(1974)
[10] Witten, E.: Nucl. Phys. **B160**, 57(1979)
[11] Sommermann, H.M., Seki, R., Larson, S., Koonin, S.E.: Phys. Rev. **D45**, 4303(1992)
[12] Shao, B., Walet, N.R., Amado, R.D.: Phys. Lett. **B 303**, 1(1993)
[13] For introduction and review of coherent states see: Klauder, J.R., Skagerstam, B-S.: Coherent States, Applications in Physics and Mathematical Physics. Singapore: World Scientific 1985.
[14] Horn, D., Silver, R.: Annals of Physics (NY) **66**, 509(1971)
[15] Botke, J.C., Scalapino, D.J., Sugar, R.L.: Phys. Rev. **D9**, 813(1974)
[16] Yang Lu, Amado, R.D.: Phys. Lett. in press; Phys. Rev. **C** in press.
[17] Amado, R.D., Cannata, F., Dedonder, J-P., Locher, M.P., Yang Lu: Phys. Lett. **B 339**, 201(1994); Phys. Rev. **C51**, 1587(1995)
[18] Yang Lu, Amado, R.D.: "Pion Correlations from Skyrmion-AntiSkyrmion Annihilation," submitted to Phys. Rev. **C**.